#### Enhanced Advanced Algebra and AP Precalculus Summer Packet 2024

I recommend completing this packet a week or two before the first day of class, so everything is fresh in your mind when beginning AP Precalculus. More than half of the questions on the AP exam do NOT allow a calculator, so please complete the no calculator questions of the summer assignment WITHOUT a calculator to be sure you are able to perform the calculations. WORK MUST BE SHOWN TO RECEIVE CREDIT.

<u>DUE DATE</u>: This summer assignment must be completed by <u>the first day of class</u>. The class will NOT go slowly to accommodate those who did not complete the summer work.

Enhanced Advanced Algebra and AP Precalculus is a high school advanced algebra course and a <u>college level</u> precalculus course all combined into one semester. Due to the large number of concepts within this course, this class will go at an <u>extremely accelerated pace</u> compared to what you've had in the past. You need to be willing to work on math in and <u>outside of class every day</u>. Please read the College Course Equivalent section below that has been extracted from the AP Precalculus Course and Exam Description written by College Board.

#### College Course Equivalent

AP Precalculus is designed to be the equivalent of a first semester college precalculus course. AP Precalculus provides students with an understanding of the concepts of college algebra, trigonometry, and additional topics that prepare students for further college-level mathematics courses. This course explores a variety of function types and their applications—polynomial, rational, exponential, logarithmic, trigonometric, polar, parametric, vector-valued, implicitly defined, and linear transformation functions using matrices. Throughout the course, the mathematical practices of procedural and symbolic fluency, multiple representations, and communication and reasoning are developed. Students experience the concepts and skills related to each function type through the lenses of modeling and covariation and engage each function type through their graphical, numerical, analytical, and verbal representations.

College Board has created a list of <u>prerequisite concepts</u> students should already know and be able to do before taking AP Precalculus. This summer assignment is designed to help you review and practice each of these prerequisite concepts. College Board has created a video playlist/series on YouTube to review each of these prerequisite concepts. Students should not find this summer assignment difficult after a quick review.

#### Link to Prerequisite Video Playlist on YouTube:

https://www.youtube.com/playlist?list=PLoGgviqq48477akYFkAyTSeTbWkF4hoWk

A graphing calculator is required for this course. Although 62.5% of the questions on the AP exam do not allow a calculator, 37.5% of the questions allow a calculator where some questions will require the use of the graphing calculator. I recommend getting a TI-83 or TI-84 graphing calculator because that is the kind I will be using and teaching with during the class, but any graphing calculator on the AP Exam approved calculator list is acceptable. For home use only, Calculate84 is a FREE app that emulates the TI-84 calculator on your phone screen. This app will not be allowed to be used in class or on the AP exam. A list of things we will teach you how to do on the graphing calculator throughout this course is listed below in the excerpt from the AP Precalculus Course and Exam Description written by College Board.

#### **Technology Needs**

Technology should be used throughout the course as a tool to explore concepts. In AP Precalculus, students should specifically practice using technology to do the following:

- Perform calculations (e.g., exponents, roots, trigonometric values, logarithms)
- Graph functions and analyze graphs
- · Generate a table of values for a function
- · Find real zeros of functions
- Find points of intersection of graphs of functions
- · Find minima/maxima of functions
- · Find numerical solutions to equations in one variable
- Find regression equations to model data (linear, quadratic, cubic, quartic, exponential, logarithmic, and sinusoidal) and plot the corresponding residuals
- Perform matrix operations (e.g., multiplication, finding inverses)

It is important to note that technology should not replace the development of symbolic manipulation skills. When algebraic expressions and equations are accessible with precalculus-level algebraic manipulation, students are expected to find zeros, solve equations, and calculate values without the help of technology. Most of the AP Exam will need to be completed without the use of technology. However, selected multiple-choice and free-response questions will require students to use a graphing calculator to complete the tasks delineated above.

#### Supplies needed for this course are the following:

- Pencils and erasers
- Graphing calculator (TI-83 or TI-84 recommended)
- 3-ring binder with loose leaf paper
- · Graph paper (optional, but helpful for graphing functions)
- Highlighters, colored pens/pencils (optional, but helpful when taking notes)

#### What You Can Expect

- Phones hang in the front of the room every day.
- Direct instruction (multiple new concepts every day no review days)
- Work bell-to-bell every day (no free days, no packing up early)
- HOMEWORK EVERY DAY
- Showing all your work a specific way and explaining your solutions in words
- Assessment approximately once each week

#### Ways to Succeed in This Class

- Participate fully in class every single day. Listen. Stay focused. Ask questions.
- Take notes every day and keep them organized by topic/date in your binder.
- Fully complete homework every day and check your solutions.
- Read your textbook and other notes/resources posted in Canvas.
- Come to tutoring before school if you need help. Use ROAR time for tutoring and practice time.
- Communicate with Ms. O'Connor! Advocate for yourself!

## Prerequisite 0 - Fraction Operations WITHOUT a Calculator

Date

Evaluate each expression. Write your answer in simplest form (no mixed numbers or decimals).

1) 
$$(-3)(\frac{1}{3})$$

2) 
$$\left(\frac{5}{7}\right)\left(\frac{1}{2}\right)\left(-\frac{9}{10}\right)$$

3) 
$$\frac{6}{5} \div \frac{-5}{6}$$

4) 
$$\frac{2}{-\frac{5}{4}}$$

5) 
$$\left(-\frac{3}{8}\right) - \left(-\frac{15}{8}\right)$$

6) 
$$\left(-\frac{5}{3}\right) - \left(-\frac{6}{5}\right)$$

7) 
$$\left(-\frac{3}{8}\right) - \frac{5}{4}$$

8) 
$$(-1) + \left(-\frac{3}{4}\right)$$

9) 
$$\frac{\left(\frac{3}{5}\right)^2}{\frac{3}{4}}$$

10) 
$$\frac{1}{2+\frac{2}{5}}$$

Use the same fraction rules you udes above to simplify the following expressions.

11) 
$$\frac{x}{3} \cdot \frac{6}{y}$$

12) 
$$\frac{x}{3} \div \frac{6}{v}$$

13) 
$$\frac{x}{3} + \frac{6}{y}$$

14) 
$$\frac{2}{3} + \frac{5}{x+6}$$

### Prerequisite 1 - Linear Functions

Date\_\_\_\_\_ Period

Write the equation of the line through the given points in a) point-slope form, b) slope-intercept form, and c) standard form.

1) through: (-5, -4) and (0, 1)

2) through: (-2, 0) and (1, 1)

slope:

slope:

point-slope:

point-slope:

slope-intercept:

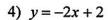
slope-intercept:

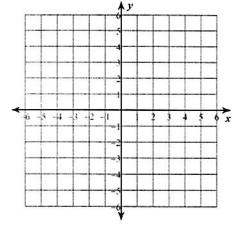
standard:

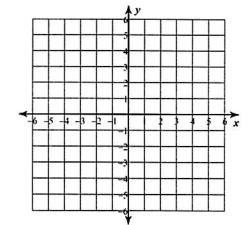
standard:

Sketch the graph of each line. Identify the x- and y-intercepts.

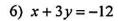
3) 
$$y = -x + 5$$

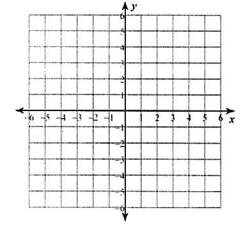


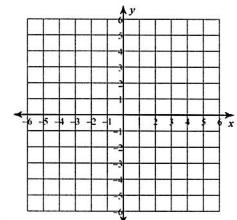




5) 
$$x - y = -1$$



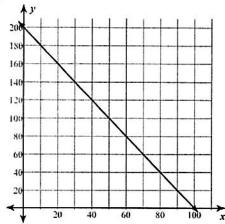




7)	Monica and Michelle rented cars from the same company. The company charges an initial fee plus a charge per mile. Monica drove 240 miles and was charge \$59.40. Michelle drove 490 miles and was charged \$74.40. Determine the:  a) What is the fee charged per mile?	8)	A school decides to sell t-shirts to raise money. If they sell 20 shirts, they will lose \$30. If they sell 100 shirts, they will make \$650. Each shirt costs the same amount. Determine the:  a) What is the cost of each shirt?
	b) What is the base fee for the rental?		b) Write a linear equation to model the situation.
	c) Write a linear equation to model the situation.		c) What is the y-intercept and what does this number mean in the context of the situation?
	d) What is the charge after 837 miles?		d) How many shirts they need to sell to break even?
	e) What is the number of miles you can drive for \$200?		e) Identify the domain and range for this function in the context of the situation.
	f) Identify the domain and range for this function in the context of the situation.		
9)	The following points are on the functoin $f(x)$ . Determine you know. $(3, 7), (5, 13), (9, 25), (12, 34)$	rmi	ne if f(x) is a linear function. Explain how

-2-

10)



Luke competes in a freestyle event, swimming at a constant pace from start to finish. The distance remaining in the race (in meters) as a function of time (in seconds) is shown in the graph. Use the graph to answer the following questions.

- a) How long did it take Luke to reach the finish? Explain your reasoning.
- b) How long (distance) was the race? Explain your reasoning.
- c) What is Luke's pace? Include units of measure.
- d) Write a linear equation to model this situation.
- e) Identify the domain and range for this function in the context of the situation.

## Prerequisite 2 - Solving Equations and Inequalities

Date\_\_\_\_\_\_ Period\_\_\_\_

Solve each equation without a calculator.

1) 
$$8-6n=-6n+8$$

2) 
$$\frac{a-1}{2a-7} = \frac{10}{7}$$

3) 
$$7(-2m+7)-6m=-11-8m$$

4) 
$$8a - 3a = -6(-5 - 3a) + 5(6a - 6)$$

5) 
$$-\frac{25}{12} = \frac{3}{2}n + \frac{3}{4} + \frac{4}{3}n$$

6) 
$$x + \frac{7}{2} + \frac{3}{8}x - \frac{11}{3} = x - \frac{1}{6}$$

Solve each inequality without a calculator and graph its solution.

7) 
$$3p - 8p > 10$$

8) 
$$11 \ge 1 - 2n - 8n$$

## Prerequisite 3 - Polynomial Addition and Multiplication

Date

Simplify each expression without a calculator.

1) 
$$3-4(x^2+2x-1)$$

2) 
$$\left(\frac{2}{3}n + n^4 - \frac{5}{8}n^3\right) + \left(\frac{8}{7}n^4 - 3 + n^3\right)$$

3) 
$$(-4.5x^3 - 1.4 - 1.7x) - (-2.3 - 3.6x + 3.9x^3)$$

3) 
$$(-4.5x^3 - 1.4 - 1.7x) - (-2.3 - 3.6x + 3.9x^3)$$
 4)  $(-6x^4 + 6x^2) + (-x^4 + 7x^2) - (-7x^2 - x^4)$ 

Find each product without a calculator.

5) 
$$(-5m-1)(-6m^2-3m+8)$$

6) 
$$(3n^2-3n+1)(-6n^2+8n-7)$$

7) 
$$(-x^3 + 4x - x^2 - 1)(2x - 3)$$

8) 
$$-2x^3(x-3x^2)(8x^2+2x-1)$$

### Prerequisite 4 - Factoring Quadratic Trinomials

Date\_\_\_\_\_Period\_\_\_

Factor each completely without a calculator. Remember to factor out the GCF first, if possible.

1) 
$$n^2 - 11n + 30$$

2) 
$$n^2 - 4$$

3) 
$$a^2 + 4a + 4$$

4) 
$$x^2 + 16x + 63$$

5) 
$$3r^2 - 21r - 54$$

6) 
$$3v^2 - 18v + 24$$

7) 
$$3n^2 + 4n - 7$$

8) 
$$3x^2 - 17x - 28$$

9) 
$$10m^2 + 29m + 21$$

10) 
$$9r^2 - 79r - 18$$

11) 
$$4n^2 - 2n - 2$$

12) 
$$60b^2 + 138b + 72$$

# Prerequisite 5 - Solving Quadratic Equations and Inequalities Date\_\_\_\_\_

Solve each equation by taking square roots without a calculator.

1) 
$$100x^2 + 8 = 33$$

2) 
$$4x^2 - 9 = 103$$

Solve each equation by factoring without a calculator.

3) 
$$1 - 2n = -n^2$$

4) 
$$2p^2 = -14 + 16p$$

5) 
$$2p^2 + 7p = -5$$

6) 
$$30 + 76v = -14v^2$$

Solve each equation using the quadratic formula without a calculator. Write your answers in exact form (no decimals).

7) 
$$36 = -n^2 - 12n$$

8) 
$$0 = 25 - p^2$$

9) 
$$x^2 + 8x = 4$$

10) 
$$-22 = -10n^2 + 10n$$

Solve each inequality without a calculator.

11) 
$$-2x^2 + 4x \le 0$$

12) 
$$2x^2 + 16x < -29$$

13) During a football game, the punter kicks the ball from the ground to the other team. The ball's height above the ground, h, in feet, after t seconds is given by the equation  $h(t) = -16t^2 + 40t$ . How long does it take for the ball to reach 20 feet above the ground? Round your answer to the nearest thousandth.

#### Prerequisite 6 - Quadratic Functions

Date\_\_\_\_\_Period

Use the information provided to write the standard form (a.k.a. general form) equation of each parabola. (Hint: Multiply and add like terms.)

1) 
$$f(x) = -20(x-7)^2 - 6$$

2) 
$$f(x) = -(x+3)^2 + 4$$

For the following functions,

A) Find the zeros algebraically (f(x) = 0). (Hint: You can factor to solve.)

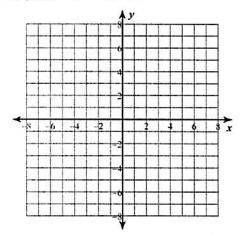
B) Identify the vertex  $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$  and tell whether it is a minimum or a maximum.

C) Determine if the parabola will open up or down.

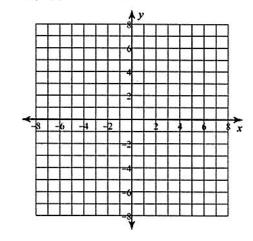
D) Sketch the graph.

E) Determine the domain and range.

3) 
$$f(x) = x^2 - 6x + 9$$



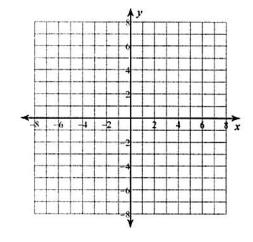
4) 
$$f(x) = -x^2 - 8x - 12$$



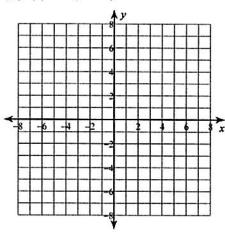
For the following functions,

- A) Find the zeros algebraically (f(x) = 0) (Hint: you can solve by square roots).
- B) Identify the vertex (h, k) and tell whether it is a minimum or a maximum.
- C) Determine if the parabola will open up or down.
- D) Sketch the graph.
- E) Determine the domain and range.

5) 
$$f(x) = \frac{3}{8}(x+2)^2 + 2$$



6) 
$$f(x) = -(x-2)^2$$



Determine if the table of values represents a quadratic function. Justify your decision.

$$7) \begin{vmatrix} x & y \\ -2 & 10 \\ -1 & 3 \\ 0 & 0 \\ 1 & 1 \\ 2 & 6 \end{vmatrix}$$

$$\begin{array}{c|cccc}
x & y \\
-2 & 7 \\
-1 & 4 \\
0 & 1 \\
1 & 4 \\
2 & 7
\end{array}$$

9) A boy tosses a penny upward as he is riding Acrophobia at Six Flags. The penny's height above the ground as a function of time can be modeled as H(t) = -16t² + 40t + 20.
a) How high is the penny when it is released? Explain your answer.

b) How long does it take the penny to reach its maximum height? Explain your answer.

c) What is the maximum height the penny reaches? Explain your answer.

d) How long is the penny in the air? Explain your answer.

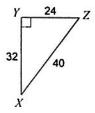
- 10) A homeowner has 40 feet of edging to place around a new, rectangular garden bed. To minimize her grocery bill, her goal is to maximize the space within the garden to grow as many vegetables as she can.
  - a) Draw and label a diagram of the situation.
  - b) Write a quadratic function in factored form to model this sutuation.
  - c) What are the zeros of the function?
  - d) Write the same quadratic function from part b) in standard (general) form.
  - e) What is the vertex of this function?
  - f) Explain the meaning of the vertex in the context of the situation.
  - g) What should the length and width of the garden be to maximize the space of the garden bed? Explain how you know.

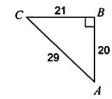
### Prerequisite 7 - Right Triangle Trigonometry

Date\_\_\_\_\_Period

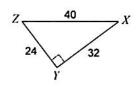
Find the value of each trigonometric ratio.

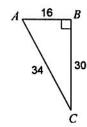
 $1) \sin Z$ 





3) 
$$\cos X$$





Find each angle measure to the nearest degree. (Make sure your calculator is in degree mode.)

5) 
$$\sin V = 0.8192$$

6) 
$$\tan A = 0.0349$$

Make sure your calculator is in degree mode for these questions.

- 7) A man is lying on the beach, flying a kite. He holds the end of the kite string at ground level and estimates the angle of elevation of the kite to be 50°. The string is 450 feet long.
  - a) Draw and label a diagram of this situation.

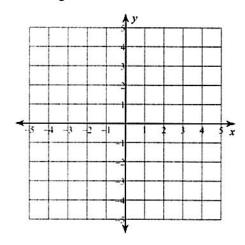
b) How high is the kite above the ground?

8)	A 96-foot tall tree casts a shadow that is 120 feet long.
	a) Draw and label a diagram of this situation.
	b) What is the angle of elevation of the sun?
9)	From a point 100 feet in front of a public library, the angles of elevation to the base of the
	flagpole and to the top of the flagpole are 28° and 39.75°, respectively. The flagpole is mounted on the roof of the library.
	a) Draw and label a diagram of this situation.
	b) Find the height of the flagpole.

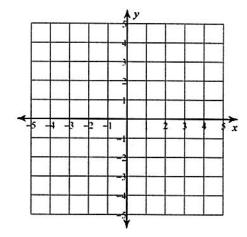
## Prerequisite 8 - Solving Systems of Equations in 2 and 3 Variables

Solve each system by graphing. No calculator.

1) 
$$y = \frac{7}{3}x + 4$$
  
 $y = \frac{2}{3}x - 1$ 



2) 
$$-1 + y = -x$$
  
 $-3 = x$ 



Solve each system by substitution without a calculator.

3) 
$$y = x - 1$$
  
 $4x - 8y = 8$ 

4) 
$$6x + 4y = -10$$
  
 $2x + y = -1$ 

Solve each system by elimination without a calculator.

5) 
$$7x + 4y = -3$$
  
 $7x + 4y = -11$ 

6) 
$$4x - 9y = 22$$
  
 $7x + 18y = -29$ 

Solve each system.

7) 
$$x + 6y + 3z = 18$$
  
 $-x + 5y + 5z = 26$   
 $x + 5y - 6z = 14$ 

8) 
$$y = x + 1$$
  
 $y = x^2 - 3x + 2$ 

#### Prerequisite 9 - Piecewise Functions

Date\_\_\_\_\_ Period\_\_

For each of the piecewise function given, evaluate the following:

- a) f(-3)
- b) f(-1)
- c) f(4)
- d) f(0)

1) 
$$f(x) = \begin{cases} -6, & x \le 0 \\ (x-2)^2, & 0 < x < 4 \\ -2, & x \ge 4 \end{cases}$$

2) 
$$f(x) = \begin{cases} -x, & x \le -3 \\ -3^x, & -3 < x < 1 \\ -5, & x \ge 1 \end{cases}$$

3) 
$$f(x) = \begin{cases} \frac{|x|}{2}, & x \le 4\\ (x-5)^4, & x > 4 \end{cases}$$

4) 
$$f(x) = \begin{cases} -2^x, & x \le -2\\ \frac{1}{x+3}, & x > -2 \end{cases}$$

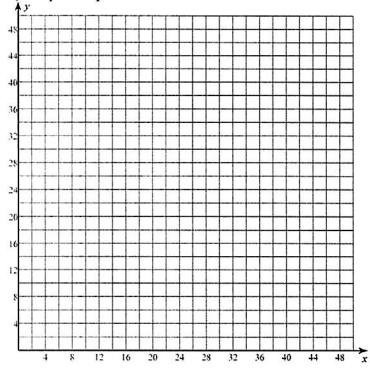
Write the following absolute value equations as a piecewise function.

5) 
$$y = |3x| - 5$$

$$6) \ \ y = \left| -\frac{1}{2}x \right| + 1$$

- 7) Shutterfly, charges \$20 for up to 20 pages of a photo book and an additional \$1 for each additional page.
  - a) Define your two variables.
  - b) Write a piecewise equation to model this situation.

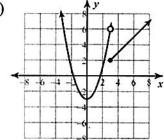
c) Graph this piecewise function.



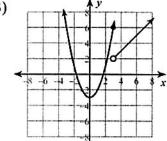
Sketch the graph of each function.

8) 
$$f(x) = \begin{cases} x^2 - 3, & x \le 3 \\ x - 1, & x > 3 \end{cases}$$

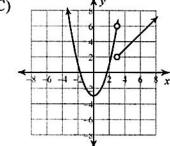




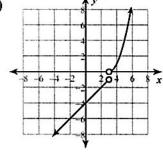
B)



C)

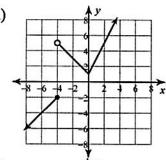


D)

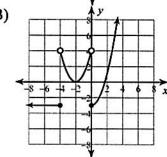


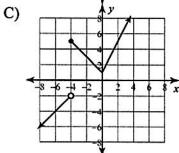
9) 
$$f(x) = \begin{cases} x+2, & x<-4\\ -x+1, & -4 \le x \le 0\\ 2x+1, & x>0 \end{cases}$$

A)

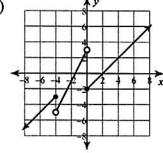


B)





D)



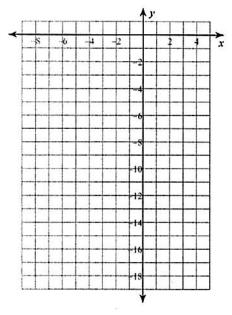
#### Prerequisite 10 - Exponential Functions

- 1) Unfortunately, the foundation of your house has about 1,200 termites. The number of termites doubles each day.
  - a) Write an equation that models the situation.
  - b) How many termites will there be after 8 days?
- 3) You have inherited land that was purchased for \$30,000 in 1960. The value of the land increased by approximately 5% per year.
  - a) Write an equation that models the situation.
  - b) What is the approximate value of the land in the year 2011?

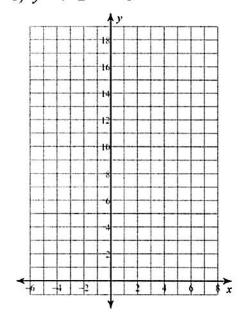
- 2) A mountain in the Rocky Mountains is eroding and losing elevation at a rate of 2.5% every year. The current elevation is 1,200 meters.
  - a) Write an equation that models the situation.
  - b) How tall will the mountain be in 2 years?
- 4) An adult takes 400 mg of medicine. Each hour, the amount of medicine in the person's system decreases by about one half.
  - a) Write an equation that models the situation.
  - b) How much of the medicine is left after 6 hours (assuming no other doses are taken)?
- 5) Elisa invests \$2,154 in a savings account with a fixed annual interest rate of 7% compounded quarterly.
  - a) Write an equation that models the situation.
  - b) What will the account balance be after 6 years?
- 6) Lea invests \$4,061 in a savings account with a fixed annual interest rate of 6.83% compounded monthly.
  - a) Write an equation that models the situation.
  - b) What will the account balance be after 10 years?

For each function, determine the a) domain, b) range, c) x-intercept(s), d) y-intercept(s), e) asymptote(s), f) end behavior, and g) where the function is increasing or decreasing. Then sketch the graph.

7) 
$$y = -2 \cdot 2^{x+2} + 1$$



8) 
$$y = 4 \cdot 2^{x-1} - 1$$



## Prerequisite 11 - Properties of Exponents

Simplify. Your answer should contain only positive exponents.

1) 
$$x^3y^{-1} \cdot 4x^0y^{-3} \cdot x^4y^{-1}$$

2) 
$$2x^{-1}y^{-2} \cdot x^{-3} \cdot x^2y^4$$

3) 
$$(4u^4v^0)^{-1}$$

4) 
$$(2a^{-4}b^4)^2$$

5) 
$$\frac{4x^3y^{-3}}{4x^2}$$

6) 
$$\frac{4x^2}{4x^2y^3}$$

7) 
$$(2u^3)^2 \cdot 2u^3 \cdot u^3v^{-2}$$

8) 
$$\frac{a^2b^{-4} \cdot 4a^4b^{-4}}{3a^4b^{-4}}$$

9) 
$$\frac{\left(2x^{-2}y^2\right)^{-2}}{\left(x^{-3}y^{-1}\right)^{-2}}$$

$$10) \; \frac{2yx^3 \cdot x}{\left(x^{-3}y^{-1}\right)^2}$$

Prerequisite 12 - Radicals (square and cube roots)

Date\_\_\_\_\_ Period

Simplify without a calculator. Rationalize denominators as needed.

1) 
$$\sqrt{75}$$

2) 
$$-4\sqrt{192}$$

3) 
$$\sqrt[3]{24}$$

4) 
$$7\sqrt[3]{216}$$

5) 
$$5\sqrt{32xy}$$

6) 
$$\sqrt[3]{-192xy^6}$$

7) 
$$-2\sqrt{2} + 3\sqrt{2}$$

8) 
$$-\sqrt{5}-\sqrt{5}$$

9) 
$$4\sqrt{6} \cdot \sqrt{3}$$

10) 
$$\sqrt{3} \cdot -2\sqrt{3}$$

11) 
$$\frac{5}{\sqrt{3}}$$

12) 
$$\frac{4\sqrt{5}}{3\sqrt{2}}$$

No calculator. Write you answer in simplest radical form.

- 13) The legs of a right triangle are 5 and 8 inches long. How long is the hypotenuse?
- 14) One of the legs of a right triangle is 5 inches long and the hypoteuse 8 inches long. How long is the other leg?

# Prerequisite 13 - Complex Numbers

Simplify.

1) 
$$(2+i)+(-5-6i)$$

2) 
$$(2-8i)+(7-7i)$$

3) 
$$(-6-2i)-(1+6i)$$

4) 
$$(-1+i)-(7-3i)$$

5) 
$$(-2-6i)^2$$

6) 
$$(-2+3i)(-6+i)$$

7) 
$$(6-4i)(2-3i)$$

8) 
$$(-4i)(-5i)(4-5i)$$

9) 
$$\frac{5}{-6i}$$

10) 
$$\frac{-3-6i}{-7i}$$

$$11) \ \frac{10i}{8+6i}$$

$$12) \ \frac{6-7i}{10+2i}$$